Model-Based Failure Detection and Isolation Scheme

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Increased interest in operational safety and economical paybacks of systems with intelligent diagnostic capabilities calls for more accurate and feasible real-time failure detection and isolation (FDI) schemes. This paper presents a model-based FDI scheme that uses state innovations to detect failures and state solution paths to isolate the failed component(s). Eigenvalue sensitivity analysis is used ot further support the determination of failed components. This analysis predicts system dynamics due to parameter changes and develops rules to more quickly isolate failed components. The resuts of this FDI scheme are applied to a fuel flow regulation system. Predictions of the FDI scheme are in very good agreement with simulation results of the artificially induced failures.

Introduction

FAILURE detection and isolation (FDI) is an important part of the overall control strategy in dynamic systems. Failures in system components can cause degradation of performance if adjustments to the control scheme cannot be made to account for these failures. FDI schemes fall into several categories based upon whether the detection scheme uses redundant hardware to detect and isolate failures or relies on software, i.e., failure detection algorithms. Reference 1 groups the methods of FDI into five categories. These categories are failure-sensitive filters, multiple-hypothesis filter detectors, voting systems, jump process formulations, and innovation-based techniques. As discussed in Refs. 1 and 2, the majority of FDI schemes in existence make use of failed or unfailed system models in detecting and isolating failures. The subject of failure detection has been considered for not only the system itself[1,2,4,5] but for those sensors providing observations from the system.[3] There are very important issues involved in the development of an FDI scheme. These include sensor and system noise, unmodeled dynamics of the system, and robustness. More recent studies have made significant contributions toward resolving these issues.[2,4,5] However, the proposed scheme usually requires a significant amount of on-line computation, especially in cases of systems with uncertainties. Such cases require determination of robust redundancy relations. Reference 2 presents a method based on an average observation subspace, which is a subset of the parity space containing a finite number of uncertain parameters. A more extended approach is given in Ref. 5.

This paper presents an FDI model-based scheme that uses state innovations for failure detection. Isolation of the failed component is performed by using state solution paths. Furthermore, for faster isolation of the failed component, this scheme utilizes eigenvalue sensitivity in highly coupled systems. Since simple failures, if not detected, can result in catastrophic failures, an FDI scheme should require short online processing time. This point has been a major focus of the FDI scheme presented in this paper.

Failure Detection Scheme

The FDI scheme is model-based and assumes the system is represented in the following state space form:

$$\dot{X} = AX + BU \tag{1}$$

$$Y = CX + DU \tag{2}$$

where $X \epsilon R^n$ is the state vector, $U \epsilon R^r$ is the input vector, $Y \epsilon R^m$ is the output vector, and the A,B,C,D matrices have the appropriate dimensions. The following assumptions are made in derivation of the FDI scheme:

1) Only failures due to system components are considered, i.e., variations in the A matrix. It is assumed no failure occurs in the way inputs are acting on the system, i.e., that there are no variations in the B matrix.

2) All system states are assumed to be available. If any state is not measurable, a fast Kalman filter is used to estimate that state.

3) Sensor failure and noise are not considered here. The treatment presented in Refs. 2, 4, and 5 can be combined with the scheme presented here to account for such terms.

4) Although system uncertainties and unmodeled dynamics are not incorporated in the present scheme, the averaged parity space approach[2] based on bounded finite uncertainties can be used to modify this scheme.

The following proposition formulates the detection process of the FDI scheme.

Proposition

Any component failure (element of the A matrix) causes only sudden abnormal variations in those state variables related to that component before the failure propagates through the whole system dynamics. Consider the kth state equation

$$\dot{x}_k = \sum_{i=1}^{n} a_{ki} x_i + \sum_{j=1}^{r} b_{kj} u_j \tag{3}$$

Deviation of a_{ki} from its normal value will only change x_k and not the other state variables. Therefore, "comparison of present values of states with their normal values (innovations) can be used to detect failures and isolate the corresponding row of the A matrix containing the failed components."

Therefore, to detect system failures, one needs to establish a set of failure thresholds for system states. These thresholds should usually be determined a priori based on experimental results. To determine those, one needs to induce artificial failures to the system, starting with small magnitudes. The values beyond which the system is pushed outside of its normal operational mode identify the failure thresholds. Once these thresholds are established, a logical statement can be formulated to identify whether or not the system is in a failure mode by monitoring the system state vector and comparing innovations with failure thresholds.

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Failed Component Isolation Scheme

Studies on the subject of FDI schemes have focused mostly on the detection part and not so much on the isolation of the failed components, which is of great interest. The isolation scheme described in this section is independent of the failure detection method. Therefore, other failure detection techniques, including those that account for noise and unmodeled dynamics,[2,4,5] can use this isolation scheme for identifying failed components. This scheme is based on solution paths for unfailed and failed states. Furthermore, to increase the chance of identifying the failed component(s), a sensitivity analysis is used.

Since an FDI scheme is eventually housed in a computer to monitor a system, the system analysis should be performed in discrete form. Consider the kth state equation of the system represented by Eq. (1):

$$x_k(K+1)=x_k(K)+\delta\sum_{j=1}^{n}a_{kj}x_j(K)+\delta\sum_{j=1}^{r}b_{kj}u_j(K) \qquad (4)$$

where δ is the time step of the discrete system.

In order to utilize solution paths, let us define a coefficient row as

$$\boldsymbol{a}_k=[a_{k1},a_{k2},\ldots,a_{k_n}] \qquad (5)$$

Then the combination of Eqs. (3) and (4) results in

$$\boldsymbol{a}_k=\left[\frac{x_k(K+1)-x_k(K)}{\delta}-\sum_{j=1}^{r}b_{kj}u_j(K)\right] \times [X^T(K)X(K)]^{-1}X^T(K) \qquad (6)$$

where T denotes the transposition of a matrix. This equation represents n algebraic equations.

Equation (6) determines entries of the A matrix based on the solution paths of state and the system inputs. This equation is used to identify the failed components. The procedure is as such: once a state variable (x_k) is detected to have abnormal behavior based on a detection scheme, the corresponding row of the A matrix ($\boldsymbol{a}_k$) will be identified. This row carries system components of which one or more has caused the failure. To isolate the failed component, the magnitude of each component is calculated by Eq. (6). By comparing the calculated values with the nominal values of these components, taking into account allowable tolerances, the failed components are identified.

This isolation scheme may get complicated by having either more than one failed component in a given row of the A matrix or multiple failures detected due to the presence of the failed component in more than one row. To resolve these types of complexity, an a priori knowledge prioritizing the main causes of failures in the system is required. Since detection and isolation schemes are based on the dynamic behavior of the system, eigenvalue sensitivity analysis would be the logical technique to obtain such knowledge.

An exact eigenvalue sensitivity analysis can be performed off-line and used to support the failed component decision. However, if the system has time-varying elements, changes due to aging, or changes due to operating conditions (linearized form of a nonlinear system), it is more appropriate to perform on-line sensitivity analysis. On the other hand, processing time for such analysis should be very short. Therefore, a simple technique for on-line eigenvalue sensitivity has to be devised. Such a scheme is outlined as follows.

Isolation Based on Eigenvalue Sensitivity

The objective of this section is to assign probabilities to components of a system (entries of A matrix) based on their effectiveness in the dynamic behavior of the system. It is shown in Ref. 5 that the trends in the relative size of the innovations due to parameter changes suggest that the system eigenvalues are more sensitive to some types of failures than to others.

For the system of Eq. (1), there is the following eigenvalue-eigenvector relationship:[7,8]

$$AV_i=\lambda_i V_i \qquad (7)$$

where V_i is the eigenvector coresponding to the λ_i eigenvalue of the A matrix. Furthermore, transposition of the A matrix (A^T) has the same eigenvalues as the A matrix but different eigenvectors, denoted by W_i. Therefore, the following relation holds:

$$A^T W_i=\lambda_i W_i$$

or

$$W_i^T A=\lambda_i W_i^T \qquad (8)$$

To study eigenvalue sensitivity with respect to changes in the elements of the A matrix $[a_{kj}]$, partial differentiation of Eq. (7) with respect to a_{kj} should be taken, which indicates

$$\frac{\partial A}{\partial a_{kj}}V_i+A\frac{\partial V_i}{\partial a_{kj}}=\frac{\partial\lambda_i}{\partial a_{kj}}V_i+\lambda_i\frac{\partial V_i}{\partial a_{kj}} \qquad (9)$$

Premultiplication of Eq. (9) by W_i^T combined with Eq. (8) results in

$$W_i^T\frac{\partial A}{\partial a_{kj}}V_i+\lambda_i W_i^T\frac{\partial V_i}{\partial a_{kj}}=W_i^T\frac{\partial\lambda_i}{\partial a_{kj}}V_i+\lambda_i W_i^T\frac{\partial V_i}{\partial a_{kj}} \qquad (10)$$

Since the matrix of eigenvectors is the inverse of the matrix of eigenrows, Eq. (10) can be simplified to Jacobi's formula:[8-10]

$$d\lambda_i=W_i^T[dA]V_i \qquad (11)$$

where $d\lambda_i$ represents changes in λ_i due to changes in the A matrix, dA. References 8 and 9 have a complete treatment of the reduction of Eq. (11) to the following form:

$$d\lambda_i=\{\mathrm{tr}[\mathrm{adjoint}(\lambda_i I-A)]\}^{-1}\{[\mathrm{adjoint}(\lambda_i I-A)]\,o\,dA\} \qquad (12)$$

where o denotes the inner product of two matrices, i.e.,

$$AoB=\Sigma a_i b_i$$

where a_i is the ith row of A and b_i is the ith column of B. For small changes in A, the following equality is held:[9]

$$[\mathrm{adjoint}(\lambda_i I-A)]\,o\,dA=\Delta[\det(\lambda_i I-A)] \qquad (13)$$

However,

$$\Delta[\det(\lambda_i I-A)]=\det(\lambda_i I-A)-\det[\lambda_i I-(A+dA)]$$

but $\det(\lambda_i I-A)$ is zero, since λ_i is an eigenvalue of A. Therefore,

$$[\mathrm{adjoint}(\lambda_i I-A)]\,o\,dA=-\det[\lambda_i-(A+dA)] \qquad (14)$$

Substitution of Eq. (14) into Eq. (12) results in

$$d\lambda_i = -\frac{\det[\lambda_i I - (A + dA)]}{\mathrm{tr}[\mathrm{adjoint}(\lambda_i I - A)]} \tag{15}$$

Equation (15) represents changes in the eigenvalues due to parameter variations in the A matrix. It should be noted that the denominator of Eq. (15) can be calculated off-line based on the original A matrix.

Define the sensitivity function as

$$S^i_{a_j} = d\lambda_i/(da_j/a_j) \tag{16}$$

The sensitivity of the ith eigenvalue with respect to the parameter a_j is represented by $S^i_{a_j}$. Since failure in one component may change all of the system eigenvalues, an average sensitivity is defined as

$$\bar{S}_{a_j} = \frac{1}{n}\sum_{i=1}^{n} \| S^i_{a_j} \|$$

or

$$\bar{S}_{a_j} = \frac{1}{n}\sum_{i=1}^{n} \left\| \frac{\det[\lambda_i I - (A + dA)]}{\mathrm{tr}[\mathrm{adjoint}(\lambda_i I - A)]} \times \frac{a_j}{da_j} \right\| \tag{17}$$

where $\|\cdot\|$ denotes the Euclidian norm.

Equation (17) reprsents an averaged sensitivity, i.e., a probability associated with every component of the system. This probability information can be used to increase the chance of isolating proper failed component(s) when the solution paths and innovations are examined.

Case Study

In order to demonstrate the application of the FDI scheme presented in this paper, failure detection and isolation in the following system is studied. Figure 1 shows a schematic diagram of a fuel flow regulator. This system provides fuel to an engine as monitored and regulated by the fuel control unit. The flow meter and tachometer provide a feedback signal for fuel flow to the control unit. The bond graph[11,12] technique is used to model this system. Figure 2 represents a detailed bond graph of the system; a significant number of system parameters are included. The state variables are f_2 (fuel volume flow rate), f_8 (rpm of the flow meter), f_{12} (current in the feedback signal), and e_6 (torque of the tachometer). The system state equations are

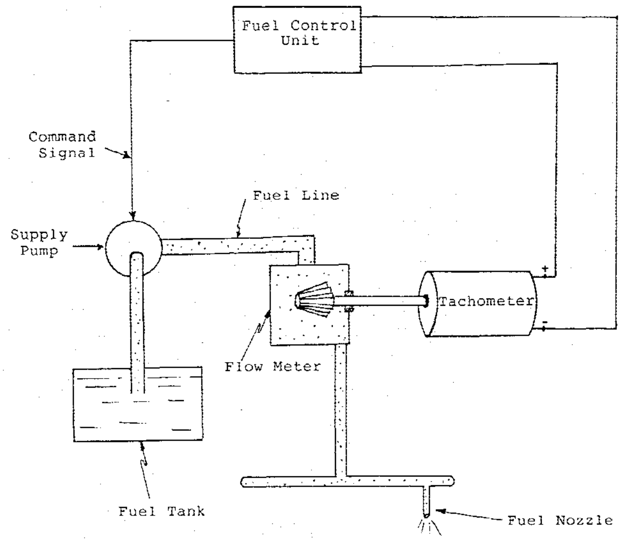
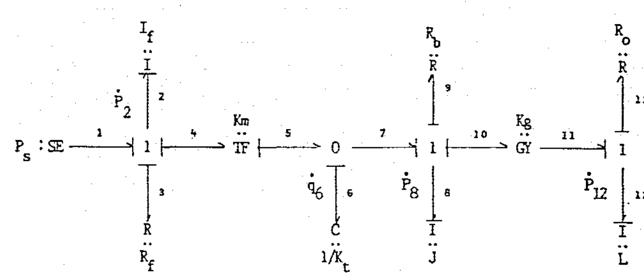


Fig. 1 Schematic of a fuel flow regular system.



PARAMETER DESCRIPTION

P_s	-	Supply Pump Pressure	R_b	-	Tachometer Bearing Friction Constant
I_f	-	Inertia of Fuel in Lines	R_o	-	Electrical Resistance of Tachometer
R_f	-	Flow Resistance of Lines	L	-	Inductance of Tachometer
Km	-	Flow Meter Torque Constant	P_2	-	Momentum of Fuel in Lines
Kt	-	Spring Constant of Tachometer Shaft	P_8	-	Angular Momentum of Tachometer Armature
J	-	Inertia of Tachometer	P_{12}	-	Flux Linkage of Inductance L
Kg	-	Tachometer Torque Constant	q_{16}	-	Torsional Deflection of Tachometer Shaft

Fig. 2 Bond graph of fuel regulator.

$$\begin{bmatrix} \dot{f}_2 \\ \dot{f}_8 \\ \dot{f}_{12} \\ \dot{e}_6 \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{I_f} & 0 & 0 & -\frac{K_m}{I_f} \\ 0 & -\frac{R_b}{J} & -\frac{K_g}{J} & \frac{1}{J} \\ 0 & \frac{K_g}{L} & -\frac{R_o}{L} & 0 \\ K_m K_T & -K_t & 0 & 0 \end{bmatrix} \begin{bmatrix} f_2 \\ f_8 \\ f_{12} \\ e_6 \end{bmatrix} + \begin{bmatrix} \frac{1}{I_f} \\ 0 \\ 0 \\ 0 \end{bmatrix} P_s$$

Based on monitoring of the state variables at a given time, the behavior shown in Fig. 3 is observed. Because of the abnormal behavior in f_8, a system failure is detected. The question is, which component has failed? To answer this question, the candidate components are first identified based on the second row of the A matrix. These components are R_b, K_g, and J. The following are steps used by the FDI scheme to isolate the failed components:

1) Based on Eq. (6), the values for the three terms $1/J$, $-K_g/J$, and $-R_b/J$ are calculated. Since the $1/J$ value does not show any changes, J is not a failed component. This reduces the choices to two, namely R_b and K_g. There is a 4.5% change in K_g/J and about a 2% change in R_b/J.

2) A common component between the second and third rows of the A matrix is K_g. Since f_{12} has shown some changes, it is likely that K_g, the tachometer, is the failed component.

3) To further support the decision that K_g is the failed component, an eigenvalue sensitivity is performed. Table 1 shows the result. It indicates that the chance of having K_g

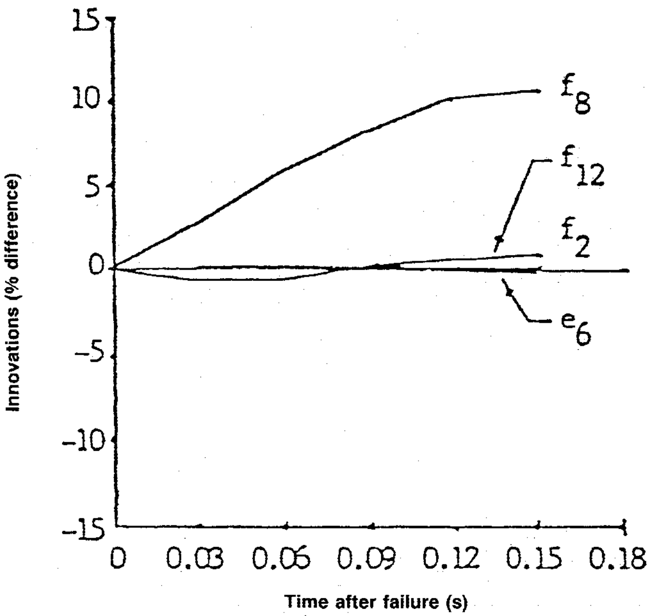


Fig. 3 Innovations for the failed system.

Table 1 Eigenvalue sensitivities to parameter changes

Failed component	$S^1_{a_j}$	$S^2_{a_j}$	$S^3_{a_j}$	$S^4_{a_j}$	S_{a_j}avg
K_g	0.2318	0.2318	0.1153	0.1153	0.6942
R_b	0.0188	0.0188	0.5158	0.5178	0.2683

fail is about three times that for R_b. Therefore, the FDI scheme predicts K_g to be the failed component.

In fact, the results shown in Fig. 3 were produced by failing K_g by 5%. This indicates that the proposed FDI scheme can predict and isolate failures accurately. More simulation results are presented in Ref. 6.

Conclusions

An on-line model-based failure detection and isolated scheme was developed. This scheme uses the innovations of states after the failure occurrence to perform failure detection and uses solution paths to identify the failed component(s). Procedures for detection and isolation processes were presented. To further support the isolation scheme results, a probability assignment was performed based on eigenvalue sensitivity to parameter variations. The sensitivity function was derived from an on-line formulation of eigenvalue variations. The resulting FDI scheme was applied to a fuel flow regulator problem. The steps that the FDI scheme would use to identify the failed components were presented.

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